

yield approximately, and to within an additive factor of  $2\pi$ ,

$$\phi_t = 2\pi \frac{Z_t}{Z_s} \left( \frac{f}{f_r} - \frac{f}{f_{r0}} \right) \quad (10)$$

where  $f_{r0}$  is the value of  $f_r$  at zero pressure. Differentiating (10), and evaluating the result at zero pressure gives

$$\frac{\partial \phi_t}{\partial P} = -2\pi \frac{Z_t}{Z_s} \frac{f}{f_{r0}} \frac{\partial \ln f_r}{\partial P} \quad (11)$$

(Although  $Z_t$  and  $Z_s$  vary with pressure, they produce only a second-order effect.) A convenient expression can be derived for the corresponding correction to the derivative of the appropriate combination of elastic moduli,  $M = \rho v^2$ . The desired phase derivative is

$$\frac{\partial \phi_s}{\partial P} = \frac{\partial \phi_m}{\partial P} - \frac{\partial \phi_t}{\partial P} \quad (12)$$

where  $\phi_m$  is the measured phase. The sample phase is  $\phi_s = 4\pi fL/v$ , so that

$$\frac{1}{\phi_s} \frac{\partial \phi_s}{\partial P} = -\frac{1}{2K} \left[ \frac{K}{M} \frac{\partial M}{\partial P} - (1 - 2K\beta) \right] \quad (13)$$

where  $\beta = -\partial \ln L / \partial P$  is the linear compressibility of the sample. The correction is, using (12),

$$\left[ \frac{\partial M}{\partial P} \right]_{\text{corr.}} = -\frac{M}{2\pi fL} \frac{\partial \phi_t}{\partial P} \quad (14)$$

Now combining (14) with (11)

$$\left[ \frac{\partial M}{\partial P} \right]_{\text{corr.}} = \frac{v^2}{L} \left( \frac{Z_t}{f_r} \frac{\partial \ln f_r}{\partial P} \right) \quad (15)$$

Thus, apart from the transducer properties, the correction to  $\partial M / \partial P$  depends only on the sample velocity and length. Some representative values are given in Table 4, for quartz transducers on  $\text{MgF}_2$  and spinel. The corrections to the pressure derivatives of the moduli are of the order of 0.1 for compressional waves and -0.05 for shear waves.

TABLE 4. Representative Corrections to Pressure Derivatives of Elastic Moduli Due to Transducer Phase Shifts

Mode	$v, \text{km/s}$	$L, \text{mm}$	$f_r, \text{Mhz}$	$\left(\frac{\partial M}{\partial P}\right)_{\text{corr.}}$	$\frac{\partial M}{\partial P}$
<u>MgF<sub>2</sub><sup>a</sup></u>					
[001]P	8.01	10.80	20	.069	5.66
[110]P	8.15	10.80	10	.141	8.43
[001]S	4.22	9.64	20	-.035	0.79
[110]S <sup>c</sup>	2.82	10.80	20	-.014	-0.68
<u>MgAl<sub>2</sub>O<sub>4</sub><sup>b</sup></u>					
[001]P	8.88	10 <sup>d</sup>	20	.091	5.15
[110]P	10.22	10	20	.120	5.85
[001]S	6.57	10	20	-.082	0.89
[110]S <sup>c</sup>	4.23	10	20	-.034	0.19

a. Davies [in preparation].

b. Chang and Barsch [1973].

c. Polarization [1 $\bar{1}$ 0].

d. Assume lengths for spinel.

Note that this difference in sign (arising from the opposite signs of the derivatives of the relevant quartz transducer frequencies [Table 3]). causes the corrections for some derived moduli to be compounded. This is most easily seen for an isometric material. If  $C_S$  denotes the modulus for a shear wave in the [110] direction, polarized in the [1 $\bar{1}$ 0] direction, then the bulk modulus is  $K = C_{11} - 4C_S/3$ , and  $C_{12} = C_{11} - 2C_S$ . The effect is illustrated in Table 5 for spinel (isometric) and MgF<sub>2</sub> (tetragonal).